Geometric and Combinatorial Separation Conditions for IFSs

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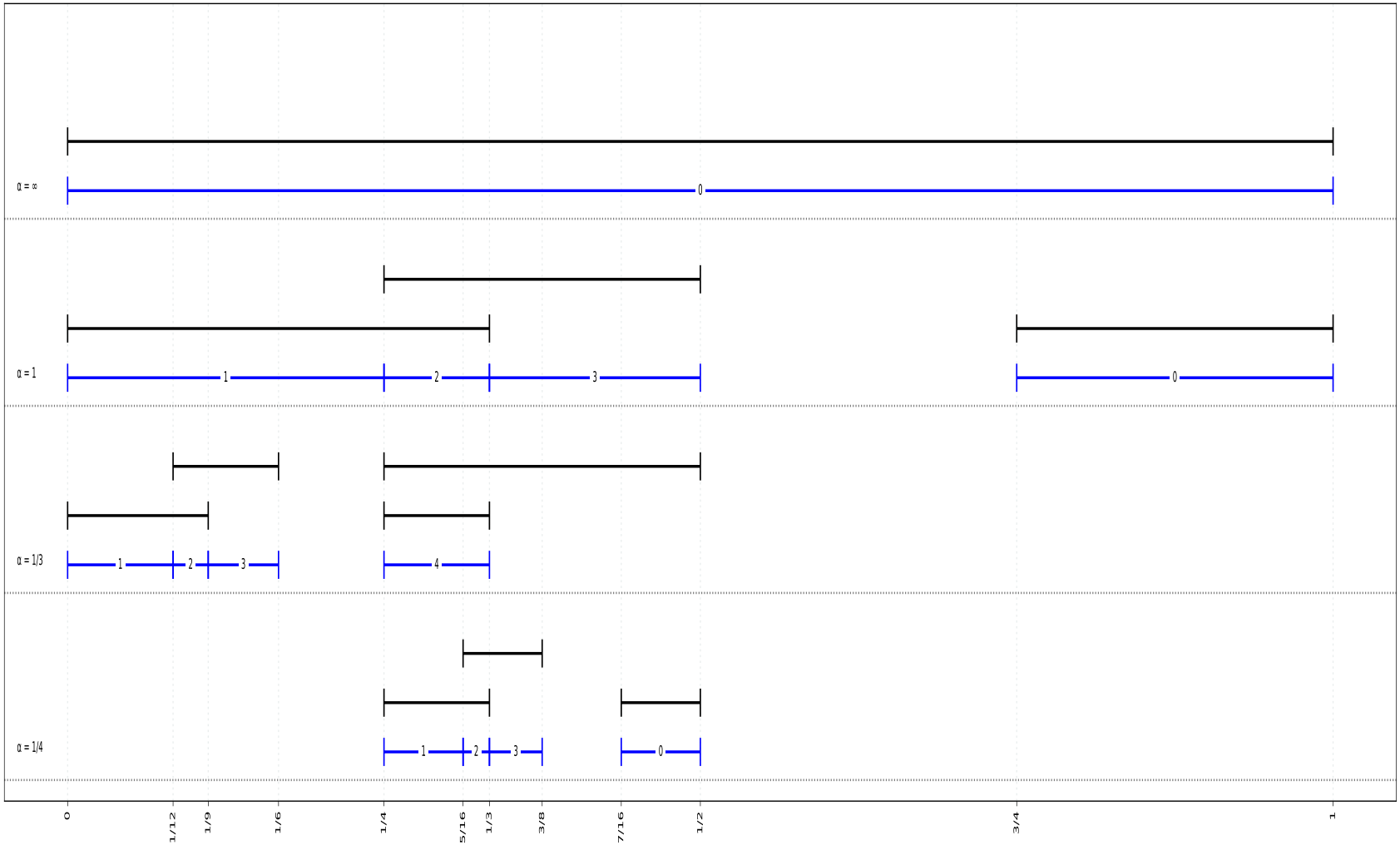
Convenient Notation

- IFS $\{S_i\}_{i=1}^k$ with $S_i(x) = r_i x + a_i$
- invariant compact set K, measure μ
- $\Sigma = \{1, \ldots, k\}$, $\Sigma^* = \{\text{finite words on } \Sigma\}$
- Given $\sigma \in \Sigma^*$, with $\sigma = (i_1, i_2, \ldots, i_n)$, write

$$S_\sigma(x) = S_{i_1} \circ \cdots \circ S_{i_n}(x)$$
$$r_\sigma = r_{i_1} \cdots r_{i_n}$$
$$\sigma^- = (i_1, i_2, \ldots, i_{n-1})$$

- Fix $\alpha > 0$; words with contraction ratio $\approx \alpha$:

$$\Lambda_\alpha = \{\sigma \in \Sigma^* : |r_\sigma| < \alpha \leq |r_{\sigma^-}|\}$$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1
2
3
0
α = 1/3
1
2
3
4
α = 1/4
1
2
3
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0
1/12
1/9
1/6
1/4
5/16
1/3
3/8
7/16
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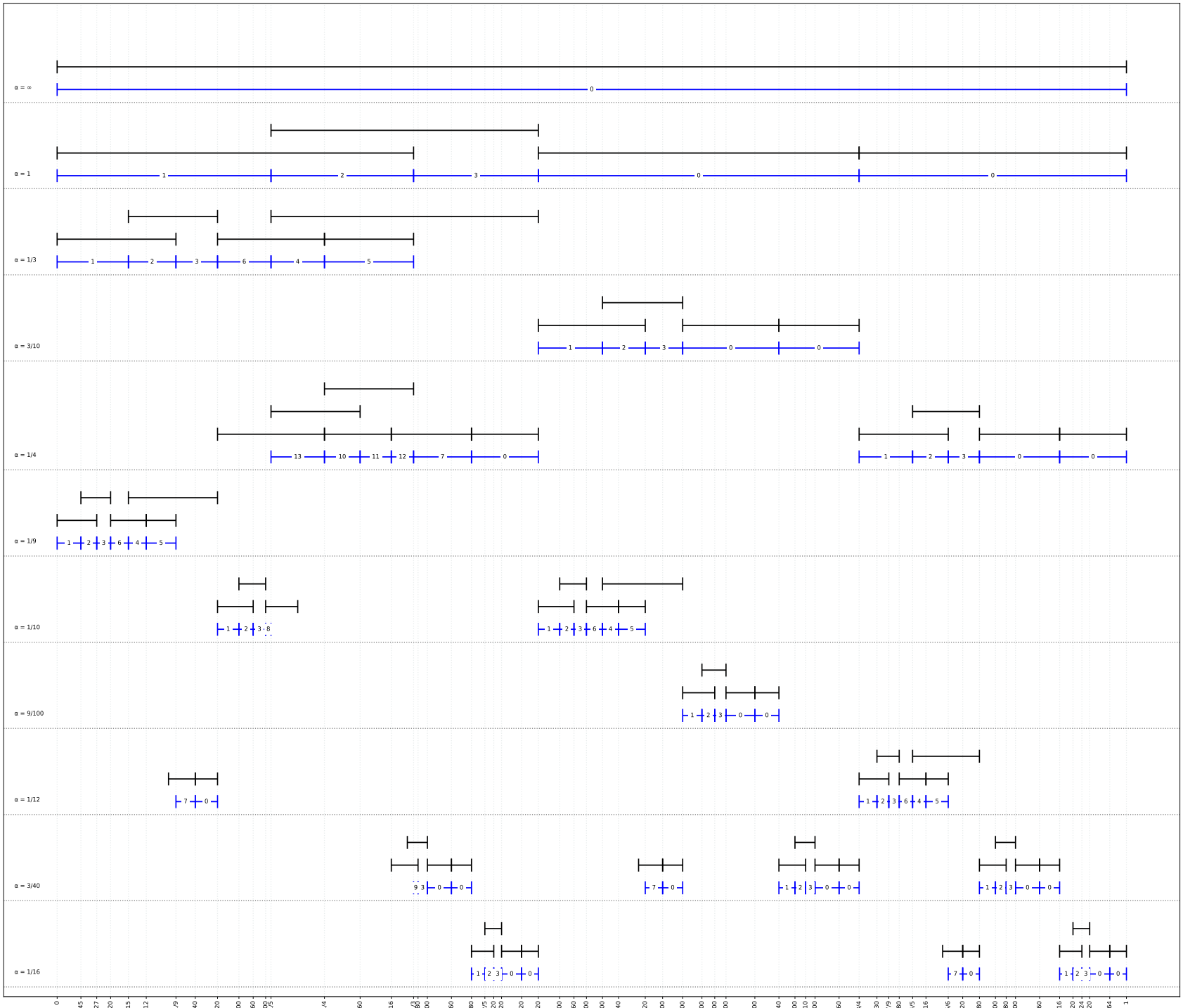
1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α = 1/16
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1/15
1/12
1/9
31/240
3/20
17/100
11/60
39/200
1/5
1/4
17/60
5/16
27/80
277/800
59/160
31/80
2/5
49/120
133/320
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47/100
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123/200
1251/2000
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27/40
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3/4
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63/80
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13/16
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717/800
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23/24
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63/64
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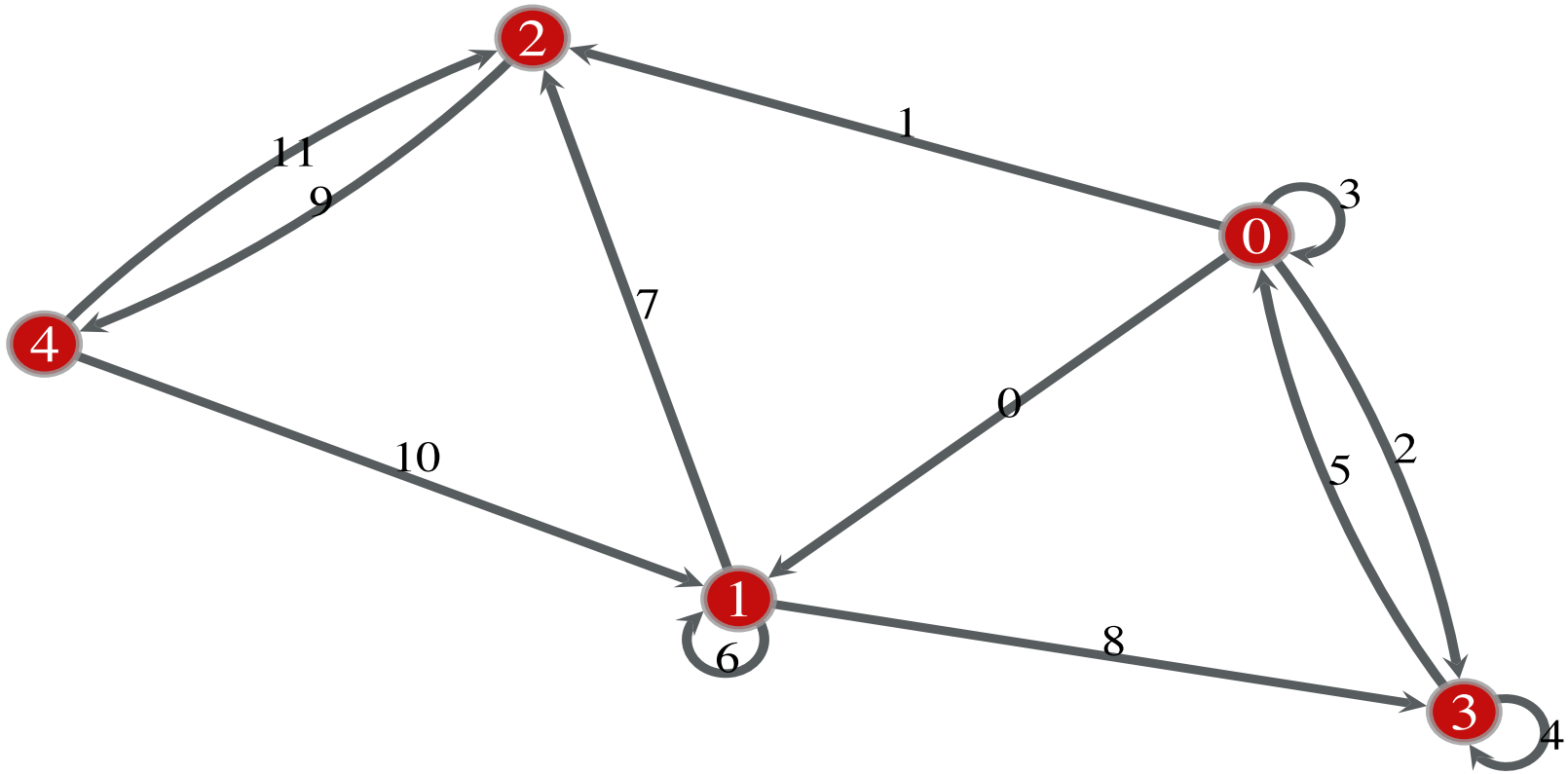
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